2.4 Projectile Motion

Sports are really science experiments in action. Consider golf balls, footballs, and tennis balls. All of these objects are projectiles (Figure 2.60). You know from personal experience that there is a relationship between the distance you can throw a ball and the angle of loft. In this section, you will learn the theory behind projectile motion and how to calculate the values you need to throw the fastball or hit the target dead on.

Try the next QuickLab and discover what factors affect the trajectory of a projectile.

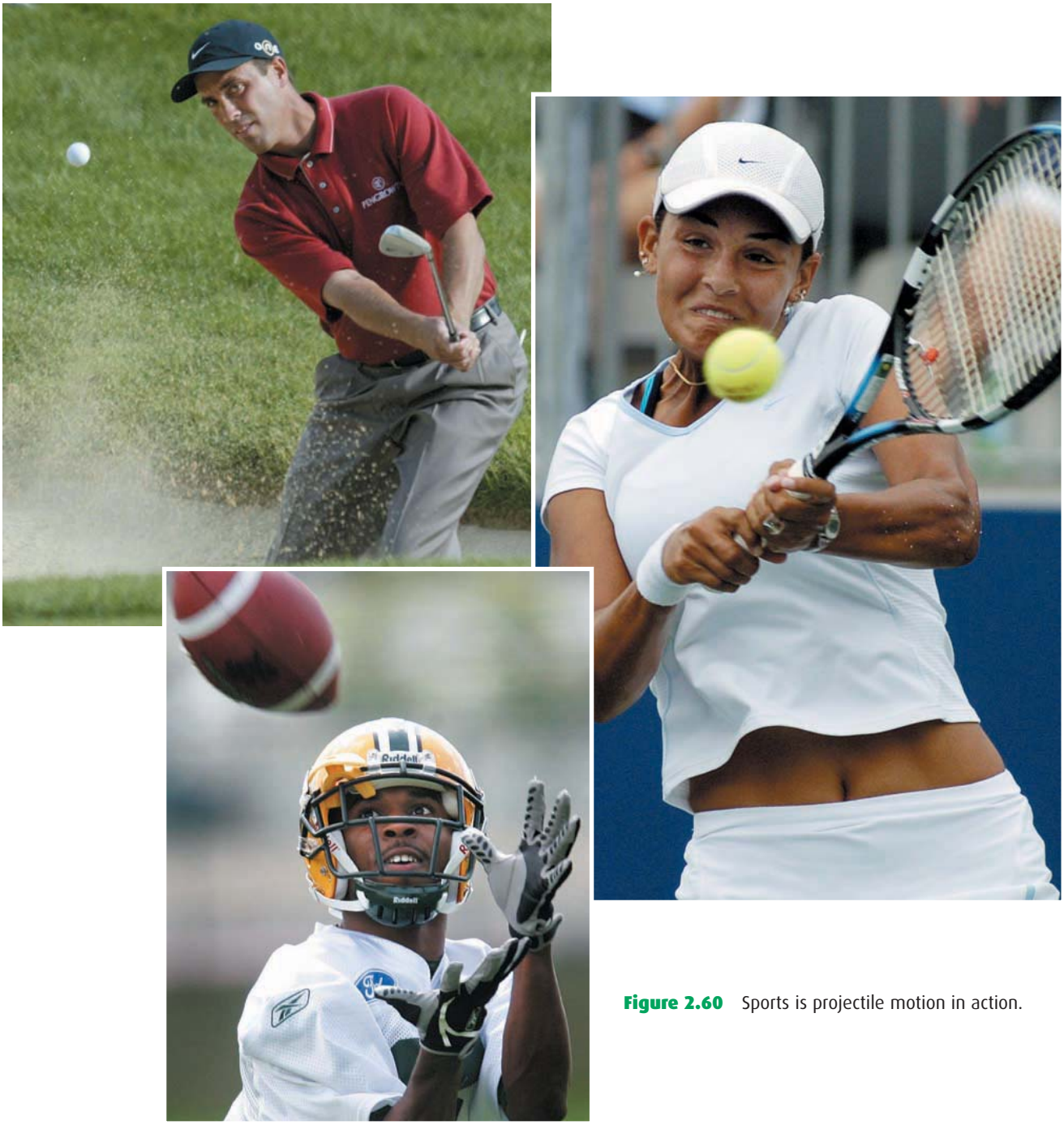


Figure 2.60 Sports is projectile motion in action.

2-4 QuickLab

Projectiles

Problem

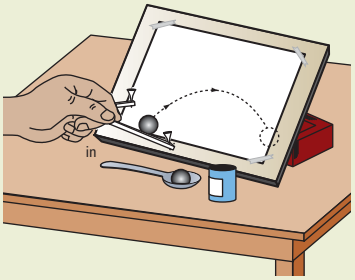
What factors affect the trajectory of a marble?

Materials

wooden board (1 m × 1 m)	two nails
hammer	elastic band
paint	spoon
marble	brick
newspaper	masking tape
white paper to cover the board	gloves

Procedure

1. Spread enough newspaper on the floor so that it covers a larger workspace than the wooden board.
2. Hammer two nails, 7.0 cm apart, at the bottom left corner of the board. Stretch the elastic between them.
3. Cover the board with white paper and affix the paper to the board using masking tape.
4. Prop the board up on the brick (Figure 2.61).
5. Wearing gloves, roll the marble in a spoonful of paint.
6. Pull the elastic band back at an angle and rest the marble in it.
7. Release the elastic band and marble. Label the marble's trajectory on the paper track 1.
8. Repeat steps 5–7 for different launch angles and extensions of the elastic band.



◀ **Figure 2.61**

Questions

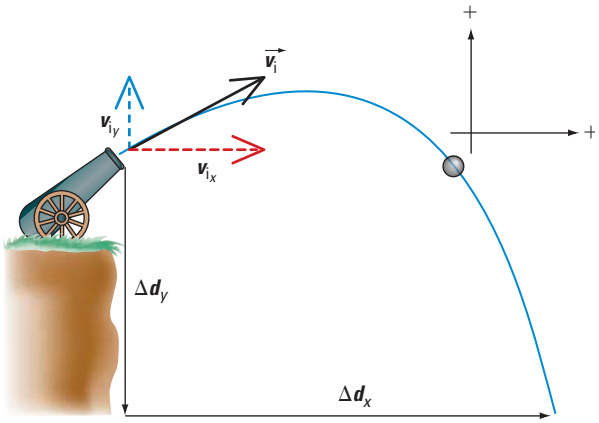
1. What is the shape of the marble's trajectory, regardless of speed and angle?
2. How did a change in the elastic band's extension affect the marble's path?
3. How did a change in launch angle affect the marble's path?

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Galileo studied projectiles and found that they moved in two directions at the same time. He determined that the motion of a projectile, neglecting air resistance, follows the curved path of a parabola. The parabolic path of a projectile is called its **trajectory** (Figure 2.62). The shape of a projectile's trajectory depends on its initial velocity — both its initial speed and direction — and on the acceleration due to gravity. To understand and analyze projectile motion, you need to consider the horizontal (x direction) and vertical (y direction) components of the object's motion separately.

trajectory: the parabolic motion of a projectile



▶ **Figure 2.62** A projectile has a parabolic trajectory.

2-5 QuickLab

Which Lands First?

Problem

What is the relationship between horizontal and vertical motion of objects on a ramp?

Materials

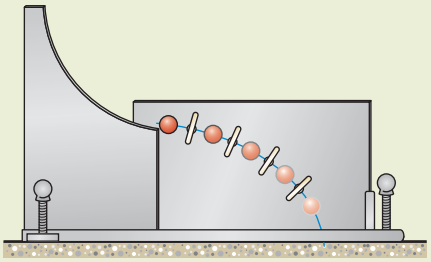
Galileo apparatus (Figure 2.63)
steel balls

Procedure

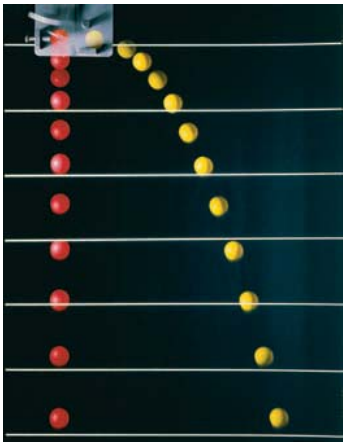
1. Set up the Galileo apparatus at the edge of a lab bench.
2. Place a steel ball at the top of each ramp.
3. Release the balls at the same time.
4. Listen for when each ball hits the ground.
5. Using a different ramp, repeat steps 1–4.

Questions

1. Which ball landed first?
2. Did the balls' initial velocity affect the result? If so, how?
3. What inference can you make about the relationship between horizontal and vertical motion?



▲ **Figure 2.63**



▲ **Figure 2.64** Gravity does not affect the horizontal motion of a projectile because perpendicular components of motion are independent.

From section 1.6, you know that gravity influences the vertical motion of a projectile by accelerating it downward. From Figure 2.64, note that gravity has no effect on an object's horizontal motion. So, the two components of a projectile's motion can be considered independently. As a result, a projectile experiences both uniform motion and uniformly accelerated motion at the same time! The *horizontal motion* of a projectile is an example of uniform motion; the projectile's horizontal velocity component is constant. The *vertical motion* of a projectile is an example of uniformly accelerated motion. The object's acceleration is the constant acceleration due to gravity or 9.81 m/s^2 [down] (neglecting friction).

Concept Check

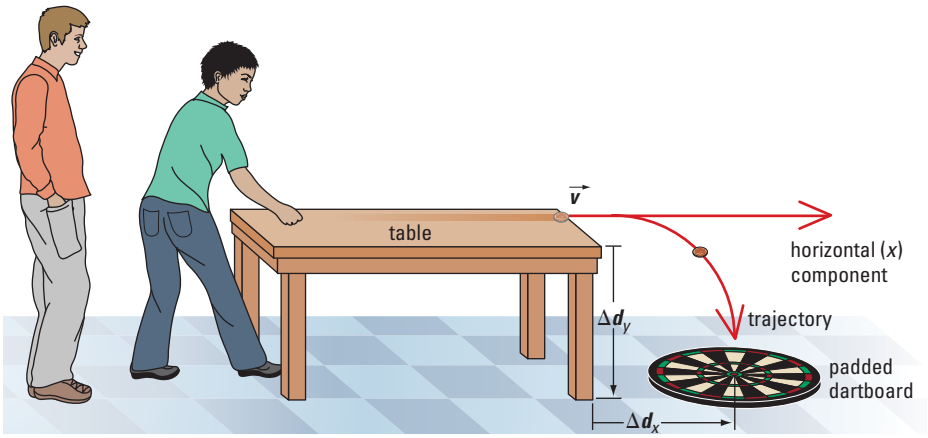
In a table, classify the horizontal and vertical components of position, velocity, and acceleration of a horizontally launched projectile as uniform or non-uniform motion.

PHYSICS INSIGHT

When a projectile is launched, for a fraction of a second, it accelerates from rest to a velocity that has x and y components.

Objects Launched Horizontally

Suppose you made a new game based on a combination of shuffleboard and darts. The goal is to flick a penny off a flat, horizontal surface, such as a tabletop, and make it land on a target similar to a dartboard beyond the table. The closer your penny lands to the bull's eye, the more points you score (Figure 2.65).



▲ **Figure 2.65** An object launched horizontally experiences uniform horizontal motion and uniformly accelerated vertical motion.

In the game, once the penny leaves the tabletop, it becomes a projectile and travels in a parabolic path toward the ground. In section 1.6, you studied motion that was caused by acceleration due to gravity. The velocity of an object falling straight down has no horizontal velocity component. In this game, the penny moves both horizontally and vertically, like the ball on the right in Figure 2.64. In this type of projectile motion, the object's initial vertical velocity is zero.

Because the projectile has a horizontal velocity component, it travels a horizontal distance along the ground from its initial launch point. This distance is called the projectile's **range** (Figure 2.66). The velocity component in the y direction increases because of the acceleration due to gravity while the x component remains the same. The combined horizontal and vertical motions produce the parabolic path of the projectile.

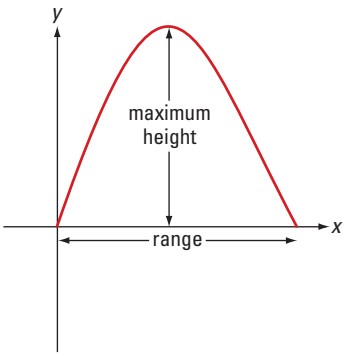
PHYSICS INSIGHT

For a projectile to have a non-zero velocity component in the vertical direction, the object must be thrown up, down, or at an angle relative to the horizontal, rather than sideways.

range: the distance a projectile travels horizontally over level ground

Concept Check

(a) What factors affecting projectile motion in the horizontal direction are being neglected?
(b) What causes the projectile to finally stop?
(c) If the projectile's initial velocity had a vertical component, would the projectile's path still be parabolic? Give reasons for your answer.

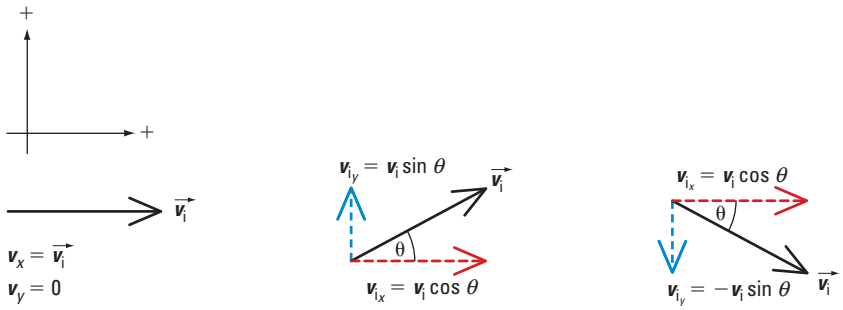


▲ **Figure 2.66** The range of a projectile is its horizontal distance travelled.

Solving Projectile Motion Problems

In this chapter, you have been working with components, so you know how to solve motion problems by breaking the motion down into its horizontal (x) and vertical (y) components.

Before you solve a projectile motion problem, review what you already know (Figure 2.67).



▲ **Figure 2.67(a)** The projectile is given an initial horizontal velocity.

▲ **Figure 2.67(b)** The projectile is given an initial horizontal velocity and an upward vertical velocity.

▲ **Figure 2.67(c)** The projectile is given an initial horizontal velocity and a downward vertical velocity.

e MATH

To explore and graph the relationship between the velocity and position of an object thrown vertically into the air, visit www.pearsoned.ca/school/physicssource.

<u>x direction</u>

- There is no acceleration in this direction, so $a_x = 0$. In this text, a_x will always be zero. The projectile undergoes uniform motion in the x direction.
- The general equation for the initial x component of the velocity can be determined using trigonometry, e.g., $v_{i_x} = v_i \cos\theta$.
- The range is Δd_x.
- Because the projectile is moving in both the horizontal and vertical directions at the same time, Δt is a common variable.

<u>y direction</u>

- If up is positive, the acceleration due to gravity is down or negative, so $a_y = -9.81\ \text{m/s}^2$.
- The y component of the initial velocity can be determined using trigonometry, e.g., $v_{i_y} = v_i \sin\theta$.
- The displacement in the y direction is Δd_y.
- Time (Δt) is the same in both the x and y directions.

▼ **Table 2.4** Projectile Problem Setup

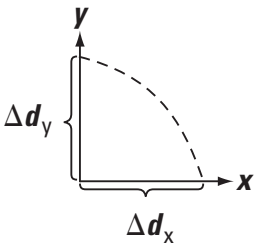
***x* direction**	***y* direction**
$a_x = 0$	$a_y = -9.81\ \text{m/s}^2$
$v_{i_x} = v_i \cos\theta$	$v_{i_y} = v_i \sin\theta$ v_{i_y} can be positive or negative depending on the direction of $\vec{v}_i$.
$\Delta d_x = v_x \Delta t$	$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

If you check the variables, you can see that they are v_i, Δt, Δd, and a, all of which are present in the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$. In the horizontal direction, the acceleration is zero, so this equation simplifies to $\Delta\vec{d} = \vec{v}_i\Delta t$. The next example shows you how to apply these equations.

Example 2.10

Head-Smashed-In Buffalo Jump, near Fort Macleod, Alberta, is a UNESCO heritage site (Figure 2.68). Over 6000 years ago, the Blackfoot people of the Plains hunted the North American bison by gathering herds and directing them over cliffs 20.0 m tall. Assuming the plain was flat so that the bison ran horizontally off the cliff, and the bison were moving at their maximum speed of 18.0 m/s at the time of the fall, determine how far from the base of the cliff the bison landed.

▲ **Figure 2.68**



◀ **Figure 2.69**

Practice Problems

1. A coin rolls off a table with an initial horizontal speed of 30 cm/s. How far will the coin land from the base of the table if the table's height is 1.25 m?
2. An arrow is fired horizontally with a speed of 25.0 m/s from the top of a 150.0-m-tall cliff. Assuming no air resistance, determine the distance the arrow will drop in 2.50 s.
3. What is the horizontal speed of an object if it lands 40.0 m away from the base of a 100-m-tall cliff?

Answers

1. 15 cm
2. 30.7 m
3. 8.86 m/s

Given

For convenience, choose forward and down to be positive because the motion is forward and down (Figure 2.69).

x direction	y direction
$v_{i_x} = 18.0$ m/s	$a_y = 9.81$ m/s^2 [down] $= +9.81$ m/s^2
	$\Delta d_y = 20.0$ m

Required

distance from the base of the cliff (Δd_x)

Analysis and Solution

Since there is no vertical component to the initial velocity of the bison, $v_{i_y} = 0$ m/s. Therefore, the bison experience uniformly accelerated motion due to gravity in the vertical direction but uniform motion in the horizontal direction resulting from the run.

From the given values, note that, in the y direction, you have all the variables except for time. So, you can solve for time in the y direction, which is the time taken to fall.

y direction:

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$= 0 + \frac{1}{2}a_y\Delta t^2$$

PHYSICS INSIGHT

For projectile motion in two dimensions, the time taken to travel horizontally equals the time taken to travel vertically.

eSIM

Analyze balls undergoing projectile motion. Follow the eSim links at www.pearsoned.ca/school/physicssource.

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{2(20.0\text{ m})}{9.81\text{ m/s}^2}}$$

$$= 2.019\text{ s}$$

x direction:
The time taken for the bison to fall vertically equals the time they travel horizontally. Substitute the value for time you found in the y direction to find the range. Since the bison had a uniform horizontal speed of 18.0 m/s, use the equation $\Delta d_x = v_{i_x}\Delta t$.

$$\Delta d_x = (18.0\text{ m/s})(2.019\text{ s})$$

$$= 36.3\text{ m}$$

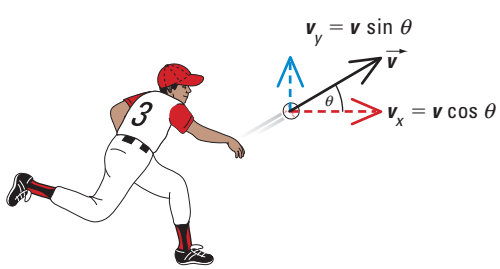
Paraphrase
The bison would land 36.3 m from the base of the cliff.

Objects Launched at an Angle

▼ **Figure 2.70** Baseball is all about projectile motion.

Baseball is a projectile game (Figure 2.70). The pitcher throws a ball at the batter, who hits it to an open area in the field. The outfielder catches the ball and throws it to second base. The runner is out. All aspects of this sequence involve projectile motion. Each sequence requires a different angle on the throw and a different speed. If the player miscalculates one of these variables, the action fails: Pitchers throw wild pitches, batters strike out, and outfielders overthrow the bases. Winning the game depends on accurately predicting the components of the initial velocity!

For objects launched at an angle, such as a baseball, the velocity of the object has both a horizontal and a vertical component. Any vector quantity can be resolved into x and y components using the trigonometric ratios $R_x = R\cos\theta$ and $R_y = R\sin\theta$, when θ is measured relative to the x-axis. To determine the horizontal and vertical components of velocity, this relationship becomes $v_x = v\cos\theta$ and $v_y = v\sin\theta$, as shown in Figure 2.71.



▲ **Figure 2.71** The horizontal and vertical components of velocity

Solving problems involving objects launched at an angle is similar to solving problems involving objects launched horizontally. The object experiences uniform motion in the horizontal direction, so use the equation $\Delta d_x = v_{i_x}\Delta t$. In the vertical direction, the object experiences uniformly accelerated motion. The general equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ still applies, but in this case, v_{i_y} is not zero. The next example shows you how to apply these equations to objects launched at an angle.

Example 2.11

Baseball players often practise their swing in a batting cage, in which a pitching machine delivers the ball (Figure 2.72). If the baseball is launched with an initial velocity of 22.0 m/s [30.0°] and the player hits it at the same height from which it was launched, for how long is the baseball in the air on its way to the batter?

▲ **Figure 2.72**

Given

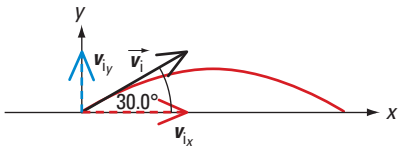
$\vec{v}_i = 22.0$ m/s [30.0°]

Required

time (Δt)

Analysis and Solution

Choose forward and up to be positive (Figure 2.73). First find the components of the baseball's initial velocity.



◀ **Figure 2.73**

x direction

$v_{i_x} = v_i \cos\theta$

$= (22.0 \text{ m/s})(\cos 30.0°)$

$= 19.05$ m/s

y direction

$v_{i_y} = v_i \sin\theta$

$= (22.0 \text{ m/s})(\sin 30.0°)$

$= 11.00$ m/s

Since the ball returns to the same height from which it was launched, $\Delta d_y = 0$. With this extra known quantity, you now have enough information in the y direction to find the time the ball spent in the air.

Practice Problems

1. A ball thrown horizontally at 10.0 m/s travels for 3.0 s before it strikes the ground. Find
   (a) the distance it travels horizontally.
   (b) the height from which it was thrown.
2. A ball is thrown with a velocity of 20.0 m/s [30°] and travels for 3.0 s before it strikes the ground. Find
   (a) the distance it travels horizontally.
   (b) the height from which it was thrown.
   (c) the maximum height of the ball.

Answers

1. (a) 30 m
   (b) 44 m
2. (a) 52 m
   (b) 14 m
   (c) 19 m

PHYSICS INSIGHT

Be careful to follow the sign convention you chose. If you chose up as positive, a_y becomes -9.81 m/s^2.

info BIT

The world's fastest bird is the peregrine falcon, with a top vertical speed of 321 km/h and a top horizontal speed of 96 km/h.

e WEB

The fastest speed for a projectile in any ball game is approximately 302 km/h in jai-alai. To learn more about jai-alai, follow the links at www.pearsoned.ca/school/physicssource.

PHYSICS INSIGHT

Since the vertical velocity of the ball at maximum height is zero, you can also calculate the time taken to go up and multiply the answer by two. If down is positive,

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{0 \text{ m/s} - (-11.00 \text{ m/s})}{9.81 \text{ m/s}^2}$$

$$= \frac{11.00 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{9.81 \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$= 1.121 \text{ s}$$

The total time the baseball is in the air is $2 \times 1.121 \text{ s} = 2.24 \text{ s}$.

info BIT

The longest speedboat jump was 36.5 m in the 1973 James Bond movie *Live and Let Die*. The boat practically flew over a road.

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$0 = (11.00 \text{ m/s})\Delta t + \frac{1}{2}(-9.81 \text{ m/s}^2)(\Delta t)^2$$

Isolate Δt and solve.

$$(4.905 \text{ m/s}^2)(\Delta t)^{\cancel{2}} = (11.00 \text{ m/s})(\cancel{\Delta t})$$

$$\Delta t = \frac{11.00 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{4.905 \frac{\text{m}}{\text{s}^2}}$$

$$= 2.24 \text{ s}$$

Paraphrase

The baseball is in the air for 2.24 s.

How far would the baseball in Example 2.11 travel horizontally if the batter missed and the baseball landed at the same height from which it was launched? Since horizontal velocity is constant,

$$\Delta d_x = v_{i_x}\Delta t$$
$$= (19.05 \text{ m/s})(2.24 \text{ s})$$
$$= 42.7 \text{ m}$$

The baseball would travel a horizontal distance of 42.7 m.

In the next example, you are given the time and are asked to solve for one of the other variables. However, the style of solving the problem remains the same. In any problem that you will be asked to solve in this course, you will always be able to solve for one quantity in either the x or y direction, and then you can substitute your answer to solve for the remaining variable(s).

Example 2.12

A paintball directed at a target is shot at an angle of 25.0°. If paint splats on its intended target at the same height from which it was launched, 3.00 s later, find the distance from the shooter to the target.

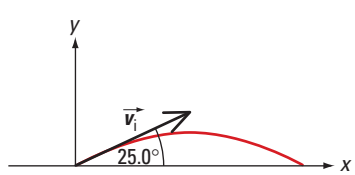
Given

Choose down and right to be positive.

$\vec{a} = a_y = 9.81 \text{ m/s}^2 \text{ [down]} = +9.81 \text{ m/s}^2$

$\theta = 25.0°$

$\Delta t = 3.00 \text{ s}$



▲ **Figure 2.74**

Required

range (Δd_x)

Analysis and Solution

Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$. Since the height of landing is the same as the launch height, $\Delta d_y = 0$.

y direction:

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$0 = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$v_{i_y}\Delta t = -\frac{1}{2}a_y(\Delta t)^2$$

$$v_{i_y} = -\frac{1}{2}a_y\Delta t$$

$$= -\frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(3.00\ \text{s})$$

$$= -14.7\ \text{m/s}$$

Since down is positive, the negative sign means that the direction of the vertical component of initial velocity is up.

x direction:

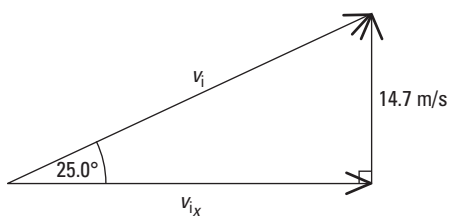
Find the initial horizontal speed using the tangent function. Because there is no acceleration in the x direction, the ball's horizontal speed remains the same during its flight: $a_x = 0$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{adjacent} = \frac{\text{opposite}}{\tan\theta}$$

$$= \frac{14.7\ \text{m/s}}{\tan 25.0°}$$

$$= 31.56\ \text{m/s}$$



▲ **Figure 2.75**

From Figure 2.75, the adjacent side is v_{i_x} and it points to the right, so $v_{i_x} = 31.56$ m/s.

Now find the horizontal distance travelled.

$$\Delta d_x = v_{i_x}\Delta t$$

$$= (31.56\ \text{m/s})(3.00\ \text{s})$$

$$= 94.7\ \text{m}$$

Paraphrase

The distance that separates the target from the shooter is 94.7 m.

PHYSICS INSIGHT

Alternatively, the time taken to reach maximum height is the same time taken to fall back down to the same height. So, the paintball is at its maximum height at 1.50 s. The speed at maximum height is zero. If up is positive,

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{v}_i = \vec{v}_f - \vec{a}\Delta t$$

$$= 0\ \text{m/s} - \vec{a}\Delta t$$

$$= -(-9.81\ \text{m/s}^2)(\Delta t)$$

$$= +(9.81\ \text{m/s}^2)(1.50\ \text{s})$$

$$= +14.7\ \text{m/s}$$

The sign is positive, so the direction is up.

Practice Problems

1. Determine the height reached by a baseball if it is released with a velocity of 17.0 m/s [20°].
2. A German U2 rocket from the Second World War had a range of 300 km, reaching a maximum height of 100 km. Determine the rocket's maximum initial velocity.

Answers

1. 1.72 m
2. 1.75×10^3 m/s [53.1°]

The points below summarize what you have learned in this section.

- To solve problems involving projectiles, first resolve the motion into its components using the trigonometric functions, then apply the kinematics equations.
- Perpendicular components of motion are independent of one another.
- Horizontal motion is considered uniform and is described by the equation $\Delta\vec{d} = \vec{v}\Delta t$, whereas vertical motion is a special case of uniformly accelerated motion, where the acceleration is the acceleration due to gravity or 9.81 m/s^2 [down].
- A projectile's path is a parabola.
- In the vertical direction, a projectile's velocity is greatest at the instant of launch and just before impact, whereas at maximum height, vertical velocity is zero.

2.4 Check and Reflect

Knowledge

1. Platform divers receive lower marks if they enter the water a distance away from the platform, whereas speed swimmers dive as far out into the pool as they can. Compare and contrast the horizontal and vertical components of each type of athlete's motion.
2. For a fixed speed, how does the range depend on the angle, θ?
3. (a) For a projectile, is there a location on its trajectory where the acceleration and velocity vectors are perpendicular? Explain.
   (b) For a projectile, is there a location on its trajectory where the acceleration and velocity vectors are parallel? Explain.
4. Water safety instructors tell novice swimmers to put their toes over the edge and jump out into the pool. Explain why, using concepts from kinematics and projectile motion.

Applications

5. Participants in a road race take water from a refreshment station and throw their empty cups away farther down the course. If a runner has a forward speed of 6.20 m/s, how far in advance of a garbage pail should he release his water cup if the vertical distance between the lid of the garbage can and the runner's point of release is 0.50 m?
6. A baseball is thrown with a velocity of 27.0 m/s [35°]. What are the components of the ball's initial velocity? How high and how far will it travel?
7. A football is thrown to a moving receiver. The football leaves the quarterback's hands 1.75 m above the ground with a velocity of 17.0 m/s [25°]. If the receiver starts 12.0 m away from the quarterback along the line of flight of the ball when it is thrown, what constant velocity must she have to get to the ball at the instant it is 1.75 m above the ground?
8. At the 2004 Olympic Games in Athens, Dwight Phillips won the gold medal in men's long jump with a jump of 8.59 m. If the angle of his jump was 23°, what was his takeoff speed?
9. A projectile is fired with an initial speed of 120 m/s at an angle of 55.0° above the horizontal from the top of a cliff 50.0 m high. Find
   (a) the time taken to reach maximum height
   (b) the maximum height with respect to the ground next to the cliff
   (c) the total time in the air
   (d) the range
   (e) the components of the final velocity just before the projectile hits the ground

Extension

10. Design a spreadsheet to determine the maximum height and range of a projectile with a launch angle that increases from 0° to 90° and whose initial speed is 20.0 m/s.

eTEST

To check your understanding of projectile motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

Key Terms and Concepts

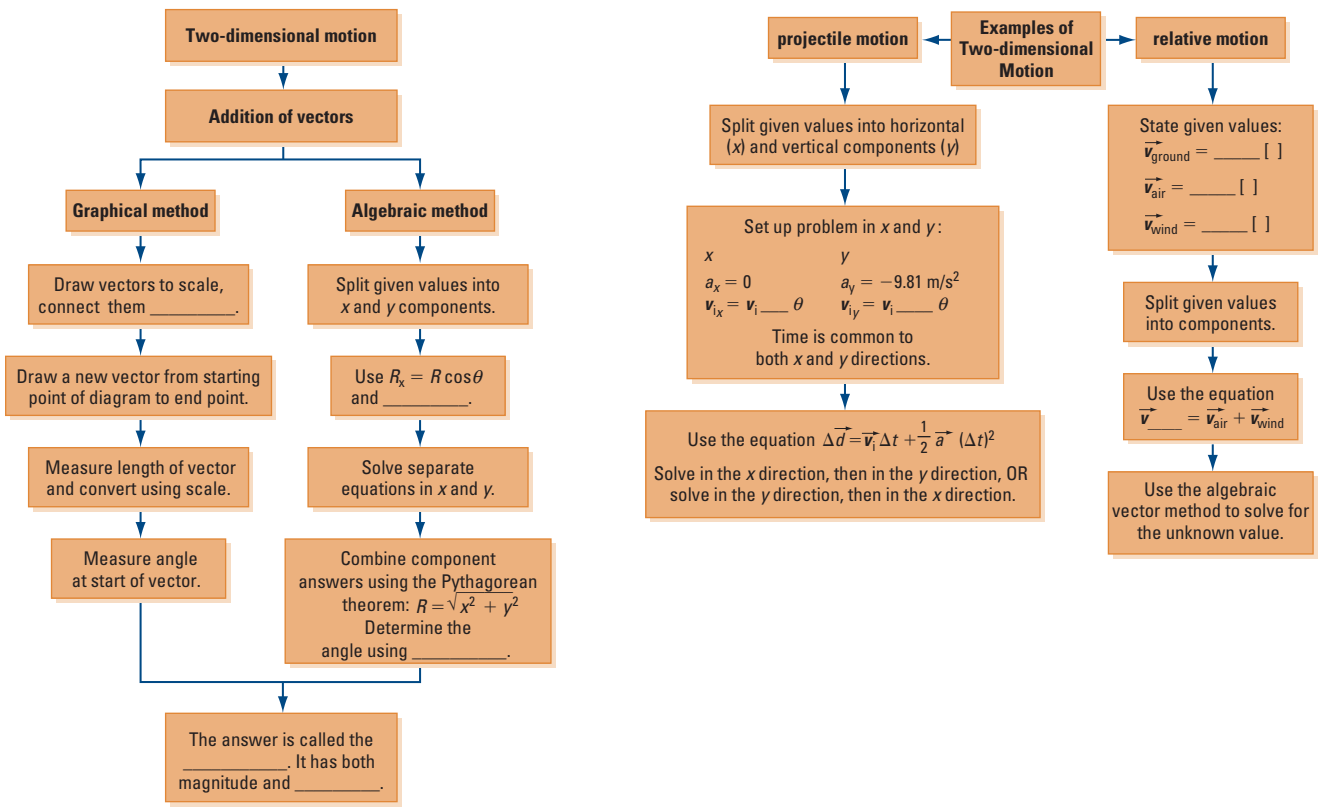
collinear	polar coordinates method	relative motion	wind velocity
resultant vector	navigator method	ground velocity	trajectory
components	non-collinear	air velocity	range

Key Equations

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2 \qquad \Delta\vec{d} = \vec{v}\Delta t$$

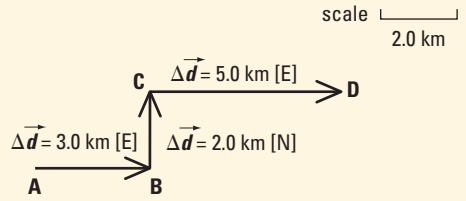
Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.



Knowledge

1. (2.2) During the Terry Fox Run, a participant travelled from A to D, passing through B and C. Copy and complete the table using the information in the diagram, a ruler calibrated in millimetres, and a protractor. In your notebook, draw and label the displacement vectors AB, BC, and CD and the position vectors AB, AC, and AD. Assume the participant's reference point is A.



	Distance Δd (km)	Final position $\vec{d}$ (km) [direction] reference point	Displacement $\Delta\vec{d}$ (km) [direction]
AB			
BC			
CD			
AC			
AD			

2. (2.2) Determine the x and y components of the displacement vector 55 m [222°].
3. (2.4) What is the vertical component for velocity at the maximum height of a projectile's trajectory?
4. (2.4) During a field goal kick, as the football rises, what is the effect on the vertical component of its velocity?
5. (2.1) Fort McMurray is approximately 500 km [N] of Edmonton. Using a scale of 1.0 cm : 50.0 km, draw a displacement vector representing this distance.
6. (2.1) Give one reason why vector diagrams must be drawn to scale.
7. (2.2) Using an appropriate scale and reference coordinates, graphically solve each of the following:
   (a) 5.0 m [S] and 10.0 m [N]
   (b) 65.0 cm [E] and 75.0 cm [E]
   (c) 1.0 km [forward] and 3.5 km [backward]
   (d) 35.0 km [right] −45.0 km [left]
8. (2.4) For an object thrown vertically upward, what is the object's initial horizontal velocity?

Applications

9. The air medivac, King Air 200, flying at 250 knots (1 knot = 1.853 km/h), makes the trip between Edmonton and Grande Prairie in 50 min. What distance does the plane travel during this time?
10. A golf ball is hit with an initial velocity of 30.0 m/s [55°]. What are the ball's range and maximum height?
11. Off the tee box, a professional golfer can drive a ball with a velocity of 80.0 m/s [10°]. How far will the ball travel horizontally before it hits the ground and for how long is the ball airborne?
12. A canoeist capable of paddling north at a speed of 4.0 m/s in still water wishes to cross a river 120 m wide. The river is flowing at 5.0 m/s [E]. Find
    (a) her velocity relative to the ground
    (b) the time it takes her to cross
13. An object is thrown horizontally off a cliff with an initial speed of 7.50 m/s. The object strikes the ground 3.0 s later. Find
    (a) the object's vertical velocity component when it reaches the ground
    (b) the distance between the base of the cliff and the object when it strikes the ground
    (c) the horizontal velocity of the object 1.50 s after its release
14. If a high jumper reaches her maximum height as she travels across the bar, determine the initial velocity she must have to clear a bar set at 2.0 m if her range during the jump is 2.0 m. What assumptions did you make to complete the calculations?
15. An alligator wishes to swim north, directly across a channel 500 m wide. There is a current of 2.0 m/s flowing east. The alligator is capable of swimming at 4.0 m/s. Find
    (a) the angle at which the alligator must point its body in order to swim directly across the channel
    (b) its velocity relative to the ground
    (c) the time it takes to cross the channel
16. A baseball player throws a ball horizontally at 45.0 m/s. How far will the ball drop before reaching first base 27.4 m away?

17. How much time can you save travelling diagonally instead of walking 750 m [N] and then 350 m [E] if your walking speed is 7.0 m/s?

18. How long will an arrow be in flight if it is shot at an angle of 25° and hits a target 50.0 m away, at the same elevation?

19. A pilot of a small plane wishes to fly west. The plane has an airspeed of 100 km/h. If there is a 30-km/h wind blowing north, find
    (a) the plane's heading
    (b) the plane's ground speed

20. At what angle was an object thrown if its initial launch speed is 15.7 m/s, it remains airborne for 2.15 s, and travels 25.0 m horizontally?

21. A coin rolls off a 25.0° incline on top of a 2.5-m-high bookcase with a speed of 30 m/s. How far from the base of the bookcase will the coin land?

22. Starting from the left end of the hockey rink, the goal line is 3.96 m to the right of the boards, the blue line is 18.29 m to the right of the goal line, the next blue line is 16.46 m to the right of the first blue line, the goal line is 18.29 m right, and the right board is 3.96 m right of the goal line. How long is a standard NHL hockey rink?

23. A plane with a ground speed of 151 km/h is moving 11° south of east. There is a wind blowing at 40 km/h, 45° south of east. Find
    (a) the plane's airspeed
    (b) the plane's heading, to the nearest degree

24. How long will a soccer ball remain in flight if it is kicked with an initial velocity of 25.0 m/s [35.0°]? How far down the field will the ball travel before it hits the ground and what will be its maximum height?

25. At what angle is an object launched if its initial vertical speed is 3.75 m/s and its initial horizontal speed is 4.50 m/s?

Extensions

26. During the Apollo 14 mission, Alan Shepard was the first person to hit a golf ball on the Moon. If a golf ball was launched from the Moon's surface with a velocity of 50 m/s [35°] and the acceleration due to gravity on the Moon is $-1.61\ m/s^2$,
    (a) how long was the golf ball in the air?
    (b) what was the golf ball's range?

27. An airplane is approaching a runway for landing. The plane's air velocity is 645 km/h [forward], moving through a headwind of 32.2 km/h. The altimeter indicates that the plane is dropping at a constant velocity of 3.0 m/s [down]. If the plane is at a height of 914.4 m and the range from the plane to the start of the runway is 45.0 km, does the pilot need to make any adjustments to her descent in order to land the plane at the start of the runway?

Consolidate Your Understanding

Create your own summary of kinematics by answering the questions below. If you want to use a graphic organizer, refer to Student References 4: Using Graphic Organizers on pp. 869–871. Use the Key Terms and Concepts listed on page 113 and the Learning Outcomes on page 68.

1. Create a flowchart to describe the different components required to analyze motion in a horizontal plane and in a vertical plane.
2. Write a paragraph describing the similarities and differences between motion in a horizontal plane and motion in a vertical plane. Share your thoughts with another classmate.

Think About It

Review your answers to the Think About It questions on page 69. How would you answer each question now?

eTEST

To check your understanding of two-dimensional motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

Are Amber Traffic Lights Timed Correctly?

Scenario

The Traffic Safety Act allows law enforcement agencies in Alberta to issue fines for violations using evidence provided by red light cameras at intersections. The cameras photograph vehicles that enter an intersection *after* the traffic lights have turned red. They record the time, date, location, violation number, and time elapsed since the light turned red. The use of red light cameras and other technology reduces the amount of speeding, running of red lights, and collisions at some intersections.

The length of time a traffic light must remain amber depends on three factors: perception time, reaction time, and braking time. The sum of perception time and reaction time is the time elapsed between the driver seeing the amber light and applying the brakes. The Ministry of Infrastructure and Transportation's (MIT) *Basic Licence Driver's Handbook* allows for a perception time of 0.75 s and a reaction time of 0.75 s. The braking time is the time it takes the vehicle to come to a full stop once the brakes are applied. Braking time depends on the vehicle's initial speed and negative acceleration. The MIT's predicted braking times are based on the assumption that vehicles travel at the posted speed limit and have a uniform acceleration of $-3.0\ m/s^2$. Other factors that affect acceleration are road conditions, vehicle and tire performance, weather conditions, and whether the vehicle was travelling up or down hill.

If drivers decide to go through an intersection safely (go distance) after a light has turned amber, they must be able to travel not only to the intersection but across it before the light turns red. The go distance depends on the speed of the vehicle, the length of the intersection, and the amount of time the light remains amber. If the driver decides to stop (stop distance), the vehicle can safely do so only if the distance from the intersection is farther than the distance travelled during perception time, reaction time, and braking time.

As part of a committee reporting to the Ministry of Infrastructure and Transportation, you must respond to concerns that drivers are being improperly fined for red light violations because of improper amber light timing. You are to decide how well the amber light time matches the posted speed limit and intersection length. Assume throughout your analysis that drivers travel at the posted speed limits.

Planning

Research or derive equations to determine

(a) a car's displacement during reaction time
(b) stop distance
(c) go distance
(d) amber light time
(e) displacement after brakes are applied
(f) amount of time elapsed after the brakes are applied

Assessing Results

After completing the project, assess its success based on a rubric* designed in class that considers

- research strategies
- experiment techniques
- clarity and thoroughness of the written report
- effectiveness of the team's presentation

Materials

- measuring tape, stopwatch

Procedure

1. Design a survey to measure the amber light times at 10 different intersections near your school. For each intersection, record its length. ***Use caution around intersections due to traffic! You may wish to estimate the length of the intersection by counting the number of steps it takes you to cross and measuring the length of your stride.***
2. Apply suitable equations to determine appropriate amber light times for the 10 different intersections.
3. Calculate stop distances and go distances for a range (±10 km/h) of posted speed limits for each intersection and plot graphs of stop distance and go distance against posted speed.

Thinking Further

1. Research the effectiveness of red light cameras in reducing accidents, speeding, and red light violations. Using your research, recommend a course of action to increase vehicle-rail safety at light-controlled railway crossings.
2. Based on your surveys and investigation, recommend whether existing amber light times should be increased, decreased, or left alone. Consider posted speeds against actual speeds and wet against dry surface conditions.
3. Prepare a presentation to the other members of your committee. Include graphs and diagrams.

*Note: Your instructor will assess the project using a similar assessment rubric.